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A NEW APPROACH TO LOW-THRUST PERTURBED TRAJECTORY OPTIMIZATION BASED ON THE USE OF COMPLEX DUAL NUMBERS

Abstract

The optimization problem of perturbed low-trajectory trajectories using the maximum principle and the homotopic method is considered, and a new approach to solving this problem based on the use of complex dual numbers is proposed. The maximum principle allows us to reduce the optimal control problem to a boundary value problem for a system of ordinary differential equations. Its application requires accurate calculation of derivatives of the Hamiltonian with respect to phase variables, which is a rather difficult task when using high-precision models of perturbing accelerations. Earlier it was proposed to use for this purpose high-precision numerical differentiation by the complex step method or automatic differentiation using dual numbers (Clifford algebra). Using these methods, the expansion of the real phase space into a complex or dual domain is required and, in fact, it is sufficient to calculate the perturbing accelerations either in the complex domain with a small increment in the imaginary part of phase variables or in the domain of dual numbers with a unit increment in the dual part of phase variables. The homotopic method reduces the boundary value problem of the maximum principle to the Cauchy problem. However, for its application, it is necessary to calculate the derivatives of the residuals of the boundary value problem with respect to the initial values of the conjugate variables, and in some cases also with other unknown trajectory parameters. The complex step method and automatic differentiation using dual numbers allows us to calculate with high accuracy only the first derivatives of differentiable functions of a real variable. However, when using the maximum principle and the homotopic method (or any other method for solving boundary value problems requiring the calculation of derivatives), the second-order mixed derivatives are actually required to be calculated. Under these conditions, the direct use of the complex step method or automatic differentiation using dual numbers is impossible. The use of multicomplex and hyperdual numbers leads to too much increase in computational costs. To overcome these circumstances, a new mathematical object is introduced - complex dual numbers, that is, dual numbers with complex coefficients. The possibility and effectiveness of their use for calculating the second-order mixed derivatives of differentiable real functions and for solving the optimization problem of perturbed low-trajectory trajectories is demonstrated. Numerical examples of optimization of low-thrust many-revolution trajectories are given taking into account the influence of the main perturbing accelerations.