IAF ASTRODYNAMICS SYMPOSIUM (C1)
Interactive Presentations - IAF ASTRODYNAMICS SYMPOSIUM (IP)

Author: Prof. Giulio Avanzini
Università del Salento, Italy, giulio.avanzini@unisalento.it

Dr. Fabrizio Giulietti
Alma Mater Studiorum - University of Bologna, Italy, fabrizio.giulietti@unibo.it
Mr. Alessandro Palmas
Nurjana Technologies, Italy, alessandro.palmas@nurjanatech.com
Mr. Danilo Zona
Università del Salento, Italy, danilo.zona@unisalento.it

# APPLICATION OF SINGULAR PERTURBATION THEORY TO SPACE FLIGHT DYNAMICS PROBLEMS

## Abstract

The objective of the work is to derive a semi-analytical approximation for orbit trajectory arcs, when, in the presence of a perturbing acceleration term, the Keplerian solution is lost. The objective is pursued by means of the application of singular perturbation theory, where the perturbation parameter, $\varepsilon$, is the ratio between the acceleration produced by the perturbing force and gravity acceleration of the primary body. Keplerian motion provides the zero-th order term of the expansion. Coefficients for first-order perturbative terms are expressed as a function of initial values of orbit parameters and perturbing parameter.

Such an approach was used in the past for deriving an analytical approximation of the orbital decay, in which case the perturbing force is represented by atmospheric drag. In the original study, density was assumed constant, hence the perturbing force is assumed simply as proportional to the square of spacecraft velocity, $V^2 \propto 1/r$, where $r$ is the radius of the orbit.

The present study aims at extending the approach in two directions. As far the orbit decay problem is concerned, a variation of density with altitude $h = r - R_\oplus$ is introduced, where $R_\oplus$ is the radius of the Earth. The second contribution is represented by the use of the same mathematical tools for the analysis of trajectory arcs under the action of a low-thrust propulsion system. In such a case the perturbing force is represented by the continuous thrusting of an electrical propulsion system, such that the perturbing force becomes constant, if sufficient electrical energy is always available. When long thrusting arcs are analyzed in the framework of heliocentric missions, the available electrical power is proportional to $1/r^2$. Both options (constant thsust force and thrust force inversely proportional to $r^2$) will be considered.

Tangential thrusting is considered and the accuracy of the perturbative expansion compared with a numerical solution obtained by a conventional integration algorithm of the equations of motion. The analysis is done for different values of the perturbing acceleration parameter, in order to identify those cases in which the approximation remains acceptable, highlighting those where, conversely, accuracy is rapidly lost.